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EFFECT OF INITIAL DEFORMATIONS ON WAVE FRONT PROPAGATION IN ANISOTROPIC PLATES

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Abstract—The effects of large amplitudes and initial deformations on shock waves and acceleration waves propagating in fiber-reinforced laminated plates are investigated. Three cases are discussed, namely the large amplitude shock under initial in-plane deformations, small amplitude waves under in-plane deformations, and small amplitude waves propagating in a plate with large deflection. It is found that the in-plane force has a substantial effect on the transverse shear mode but little effects on other modes. The large initial deflection, however, is found to have considerable effects on all modes. A general procedure for constructing the wave surfaces is also presented.

1. INTRODUCTION

In a previous paper[1], the present authors developed a set of general equations on the shock wave and the acceleration wave propagating with large amplitude in initially undisturbed and disturbed laminated plates. Discussions were given only on the case without initial deformations. In[1], it was found that the acceleration wave could be affected only by the initial deformation immediately ahead of the wave front. It was also revealed that large amplitudes had little influence on the propagation of all wave fronts of all modes except the transverse shear mode if the plate was initially undisturbed. It is thus of interest to investigate the effect of initial deflection of the laminated plate on the propagation of wave fronts.

In this paper, we employ the basic governing equations derived in[1] to study several specific initial deformations. A general method for constructing the wave surfaces is also presented.

2. GOVERNING EQUATIONS

The notations will follow those used by[1].

It was derived in [1] that the plate kinematical variables u_i of the shock wave in a laminate initially at rest or in motion must satisfy the relation

$$\{a_{ij}\}\{\dot{u}_j\} = \{0\} \tag{1}$$

where $\{a_{ij}\}$ and $\{u_i\}$ are given in[1]. The corresponding relation for the acceleration wave was obtained as

$$\{b_{ij}\}\{\ddot{u}_j\} = \{0\} \tag{2}$$

The conditions that the velocities of the shock wave and the acceleration wave must satisfy are

$$|a_{ij}| = 0 \tag{3}$$

and

$$|b_{ij}| = 0 \tag{4}$$

respectively. Specifically, the general expression for $|a_{ij}| = 0$ is given by

$$\begin{vmatrix} A^*_{11}-c_n^2P & A^*_{12} & B^*_{11}-c_n^2R & B^*_{12} & k_1^0-\frac{1}{2}k_1\frac{[\dot w]}{c_n} \\ A^*_{21} & A^*_{22}-c_n^2P & B^*_{21} & B^*_{22}-c_n^2R & k_2^0-\frac{1}{2}k_2\frac{[\dot w]}{c_n} \\ B^*_{11}-c_n^2R & B^*_{12} & D^*_{11}-c_n^2I & D^*_{12} & k_3^0-\frac{1}{2}k_3\frac{[\dot w]}{c_n} \\ B^*_{21} & B^*_{22}-c_n^2R & D^*_{21} & D^*_{22}-c_n^2I & k_4^0-\frac{1}{2}k_4\frac{[\dot w]}{c_n} \\ k_1^0-k_1\frac{[\dot w]}{c_n} & k_2^0-k_2\frac{[\dot w]}{c_n} & k_3^0-k_3\frac{[\dot w]}{c_n} & k_4^0-k_4\frac{[\dot w]}{c_n} & k_5-c_n^2P+\delta_n \end{vmatrix} = 0 \qquad (5)$$

where

$$k_5 = n_x^2A_{55}+n_y^2A_{44}+2n_xn_yA_{45} \qquad (6)$$

$$\{k\} = \begin{Bmatrix} \{A^*\} \\ \{B^*\} \end{Bmatrix}\{n\} \qquad (7)$$

$$\{k^0\} = \begin{Bmatrix} \{A^*\} \\ \{B^*\} \end{Bmatrix}\{E_a\} \qquad (8)$$

$$\delta_n = \gamma_0 - 3\gamma_1^0[\dot w]/2c_n + \gamma[\dot w]^2/2c_n^2 \qquad (9)$$

$$\gamma = \{n\}^T\{A^*\}\{n\} \qquad (10)$$

$$\gamma_1^0 = \{n\}^T\{A^*\}\{E_a\} \qquad (11)$$

$$\gamma_0 = \{n\}^T\{T\}^T\{N_a\} + \{E_a\}^T\{A^*\}\{E_a\} \qquad (12)$$

and

$$\{E_a\} = \begin{Bmatrix} \left(\dfrac{\partial w}{\partial x}\right)_a \\ \left(\dfrac{\partial w}{\partial y}\right)_a \end{Bmatrix}. \qquad (13a)$$

$$\{N_a\} = \{A\}\begin{Bmatrix} \left(\dfrac{\partial u}{\partial x}\right)_a \\ \left(\dfrac{\partial v}{\partial y}\right)_a \\ \left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)_a \end{Bmatrix} + \{B\}\begin{Bmatrix} \left(\dfrac{\partial \psi_x}{\partial x}\right)_a \\ \left(\dfrac{\partial \psi_y}{\partial y}\right)_a \\ \left(\dfrac{\partial \psi_x}{\partial y}+\dfrac{\partial \psi_y}{\partial x}\right)_a \end{Bmatrix} + \frac{1}{2}\{A\}\begin{Bmatrix} \left(\dfrac{\partial w}{\partial x}\right)_a^2 \\ \left(\dfrac{\partial w}{\partial y}\right)_a^2 \\ 2\left(\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\right)_a \end{Bmatrix} \qquad (13b)$$

The definitions of other quantities are given by [1]. In the above equations, a subscript "a" again designates the value immediately ahead of the wave front. It is important to note that the influence of the initial deformation on the wave front is through the nontrivial terms $\{E_a\}$ and $\{N_a\}$ existing immediately ahead of the wave front.

The expression for $\{b_0\}$ can be obtained from eqn (5) by setting $[\dot w] = 0$.

3. LARGE AMPLITUDE SHOCK UNDER INITIAL IN-PLANE DEFORMATION

Suppose that the laminated plate is in a state of in-plane deformation so that the transverse displacement vanishes everywhere. Consequently,

$$\{E_a\} = \{0\},\ \{k^0\} = \{0\},\ \gamma_1^0 = 0 \qquad (14)$$

and

$$\gamma_0 = \{n\}^T\{T\}^T\{N_a\} \qquad (15)$$

A careful examination of eqn (15) reveals that γ_0 is in fact the normal component of the initial in-plane force, i.e.

$$\gamma_0 = (N_\nu)_a = n_x^2(N_x)_a + n_y^2(N_y)_a + 2n_x n_y (N_{xy})_a \tag{16}$$

Substituting eqns (14) and (16) in eqn (9) we obtain

$$\delta_n = (N_\nu)_a + \gamma[\dot{w}]^2/2c_n^2 \tag{17}$$

Substitution of eqns (14) and (17) in eqn (5) yields, for $R = 0$,

$$\begin{vmatrix} A_{11}^* - c_n^2P & A_{12}^* & B_{11}^* & B_{12}^* & k_1[\dot{w}] \\ A_{21}^* & A_{22}^* - c_n^2P & B_{21}^* & B_{22}^* & k_2[\dot{w}] \\ B_{11}^* & B_{12}^* & D_{11}^* - c_n^2I & D_{12}^* & k_3[\dot{w}] \\ B_{21}^* & B_{22}^* & D_{21}^* & D_{22}^* - c_n^2I & k_4[\dot{w}] \\ k_1[\dot{w}] & k_2[\dot{w}] & k_3[\dot{w}] & k_4[\dot{w}] & 2c_n^2(k_5^s - c_n^2P) + \gamma[\dot{w}]^2 \end{vmatrix} = 0 \tag{18}$$

where

$$\begin{aligned} k_5^s &= k_5 + (N_\nu)_a \\ &= n_x^2(N_x + A_{55})_a + n_y^2(N_y + A_{44})_a + 2n_x n_y (N_{xy} + A_{45})_a \end{aligned} \tag{19}$$

It is easy to see that, except for the term k_5^s, the expression $|a_{ij}| = 0$ given by eqn (18) is identical to that obtained in [1] for the case of initially undisturbed plates. The effects of the initial stresses are entirely absorbed by the quantity k_5^s. It is noted that if the initial in-plane forces are positive, then these would be a stiffening effect.

It was discussed in [1] that one of the roots of the determinantal eqn (18) is trivial. Thus, there are five roots for c_n^2 associated with five wave fronts. Also, in order that a transverse shear shock can propagate, the following inequality must be satisfied:

$$[w]^2 \leq 2(k_5^s)_{\min} \tag{20}$$

where $(k_5^s)_{\min}$ is the minimum value of k_5^s for all possible propagation directions. This direction can be obtained by setting the derivative of k_5^s given by eqn (19) with respect to θ (the angle between the wave front normal and the x-axis) to zero. We obtain

$$\tan 2\theta_{\min} = 2(N_{xy} + A_{45})/(N_x - N_y + A_{55} - A_{44}) \tag{21}$$

The corresponding minimum value of k_5^s is

$$2(k_5^s)_{\min} = N_x + N_y + A_{55} + A_{44} - \{(N_x - N_y + A_{55} - A_{44})^2 + 4(N_{xy} + A_{45})^2\}^{1/2} \tag{22}$$

The square root of this value is then the maximum value of the amplitude with which the shock wave can propagate in any direction. In other words, the shock wave would cease to propagate in the direction $\theta_{\min}$ if $[\dot{w}]^2$ reaches the value equal to $2(k_5^s)_{\min}$.

4. SMALL AMPLITUDE WAVES UNDER IN-PLANE DEFORMATIONS

This is a special case of the one discussed previously. If the amplitude of the shock front is small, then we set

$$[\dot{w}] = 0 \tag{23}$$

in eqn (18) to obtain

$$\begin{vmatrix} A_{11}^* - c_n^2P & A_{12}^* & B_{11}^* & B_{12}^* & 0 \\ A_{21}^* & A_{22}^* - c_n^2P & B_{21}^* & B_{22}^* & 0 \\ B_{11}^* & B_{12}^* & D_{11}^* - c_n^2I & D_{12}^* & 0 \\ B_{21}^* & B_{22}^* & D_{21}^* & D_{22}^* - c_n^2I & 0 \\ 0 & 0 & 0 & 0 & k_5^s - c_n^2P \end{vmatrix} = 0 \tag{24}$$

It should be pointed out here that the velocity for the acceleration wave propagating under the same conditions satisfies the same equation for the shock wave as given by eqn (24). In fact, for acceleration waves with large amplitudes, the condition $|b_{ij}| = 0$ assumes the same expression of eqn (24).

From eqn (24), it can be seen that there is only one wave front that is affected by the initial in-plane forces. This wave front can be easily identified to be the transverse shear wave for which the normal velocity is given by

$$Pc_n^2 = k_5^0 \tag{25}$$

Explicitly, eqn (25) can be expressed as

$$Pc_n^2 = \frac{1}{2}(N_x + N_y + A_{55} + A_{44})_a + \frac{1}{2}(N_x - N_y + A_{55} - A_{44})_a \cos 2\theta + (N_{xy} + A_{45})_a \sin 2\theta \tag{26}$$

In view of eqn (26), we conclude that the wave front can propagate with a constant velocity independent of direction if the initial deformation is set up in such a way that

$$N_{xy} + A_{45} = 0, \quad N_x + A_{55} = N_y + A_{44} \tag{27}$$

5. EFFECT OF LARGE INITIAL DEFLECTION ON SMALL AMPLITUDE WAVES

If the amplitude of the wave front is small then we obtain from eqn (9)

$$\delta_n = \gamma_0 \tag{28}$$

Substituting eqn (28) together with $[\dot{w}] = 0$ in eqn (5) we obtain

$$\begin{vmatrix} A_{11}^* - c_n^2 P & A_{12}^* & B_{11}^* & B_{12}^* & k_1^0 \\ A_{21}^* & A_{22}^* - c_n^2 P & B_{21}^* & B_{22}^* & k_2^0 \\ B_{11}^* & B_{12}^* & D_{11}^* - c_n^2 I & D_{12}^* & k_3^0 \\ B_{21}^* & B_{22}^* & D_{21}^* & D_{22}^* - c_n^2 I & k_4^0 \\ k_1^0 & k_2^0 & k_3^0 & k_4^0 & k_5 + \gamma_0 - c_n^2 P \end{vmatrix} = 0 \tag{29}$$

In eqn (29), the quantities $\{k^0\}$ and $\{\gamma^0\}$ depend on $\{E_a\}$ and $\{N_a\}$ which have to be determined from the given initial deformation of the laminated plate. In the following, we will consider the simple initial deflection given by

$$w_a(x, y) = d_0(x \cos \theta_0 + y \sin \theta_0) = d_0 \eta \tag{30}$$

The state of deformation of the plate is not completely defined by eqn (30), since the other displacement components and the initial static loadings still can vary.

Consider a state of deformation that is obtained by simply moving the edge of the plate along $\eta = l$ vertically. This results in a stretching effect. If the external transverse loads are vanishing, then the transverse shear deformations can be eliminated, i.e.

$$\psi_{xa} + \frac{\partial w_a}{\partial x} = 0, \quad \psi_{ya} + \frac{\partial w_a}{\partial y} = 0 \tag{31}$$

Substituting eqn (30) in eqn (31), we obtain

$$\psi_{xa} = -d_0 \cos \theta_0, \quad \psi_{ya} = -d_0 \sin \theta_0 \tag{32}$$

If, in addition, we require that

$$u_a = v_a = 0 \tag{33}$$

then the static equations of equilibrium of a laminated plate under large deflection can be satisfied identically. The nonlinear equations of equilibrium can be found in[2]. To realize the boundary conditions for a plate of width l in the η-direction proves to be easy. It can be shown that the state of deformation given by eqns (30), (32) and (33) can be obtained by introducing hinged edge conditions along $\eta = 0$ and $\eta = l$, with externally applied edge moments given by

$$\{M\} = \frac{1}{2} d_0^2 \{B\}\{T_0\}\{n_0\} \tag{34}$$

where

$$\{T_0\} = \begin{Bmatrix} n_{x0} & 0 \\ 0 & n_{y0} \\ n_{y0} & n_{x0} \end{Bmatrix}, \qquad \{n_0\} = \begin{Bmatrix} n_{x0} \\ n_{y0} \end{Bmatrix} \tag{35}$$

with $n_{x0} = \cos\theta_0$ and $n_{y0} = \sin\theta_0$, together with an imposed displacement $w = d_0 l$ at $\eta = l$. For symmetric laminates, $\{B\} = 0$ and, thus, $\{M\} = 0$.

Substitution of eqn (30) in eqn (13) yields

$$\{N_a\} = \frac{1}{2} d_0^2 \{A\}\{T_0\}\{n_0\}, \tag{36a}$$

and

$$\{E_a\} = d_0\{n_0\} \tag{36b}$$

respectively. Using eqns (36a) and (36b), we obtain from eqns (12) and (8)

$$\gamma_0 = d_0^2\left(\{n_0\}^T\{A^*\}\{n_0\} + \frac{1}{2}\{n\}^T\{T\}^T\{A\}\{T_0\}\{n_0\}\right) \tag{37a}$$

and

$$\{k^0\} = d_0 \begin{Bmatrix} \{A^*\} \\ \{B^*\} \end{Bmatrix} \{n_0\}, \tag{37b}$$

respectively. With γ_0 and $\{k^0\}$ determined in terms of the given deflection, the eigen-value problem given by eqn (29) can be solved as usual. The numerical solutions will be presented in a later section.

6. CONSTRUCTION OF THE WAVE SURFACE

Assume that the wave front position is given by

$$f(x, y) = t \tag{38}$$

for a given time t. By defining

$$p_x = \partial f/\partial x, \quad p_y = \partial f/\partial y \tag{39}$$

we obtain from eqn (38)

$$\frac{df}{dt} = p_x\frac{dx}{dt} + p_y\frac{dy}{dt} = 1 \tag{40}$$

The quantities p_x and p_y are, in fact, components of the normal slowness vector.

The normal vector to the wave front is given by

$$n_x = p_x/(p_x^2+p_y^2)^{1/2}, \quad n_y = p_y/(p_x^2+p_y^2)^{1/2} \tag{41}$$

From eqns (40) and (41), it is easy to show that

$$c_n = 1/(p_x^2+p_y^2)^{1/2} \tag{42}$$

It is then obvious that

$$n_x = c_n p_x, \quad n_y = c_n p_y \tag{43}$$

Using the above relations, the determinantal equation $|a_{ij}| = 0$ or $|b_{ij}| = 0$ can be expressed in terms of p_x and p_y. The general equation for the normal velocity c_n can be written as

$$g(p_x, p_y) = 0 \tag{44}$$

It is noted that eqn (44) represents five equations corresponding to the five values of c_n. Thus, we have five equations characterizing the five propagating wave fronts. In view of the relations given by eqn (39), we may regard each equation $g(p_x, p_y) = 0$ corresponding to each value of c_n as a first order partial differential equation in f. The positions of the wave fronts can be obtained by solving these nonlinear partial differential equations. Such solutions can be obtained by reducing the partial differential equation to a family of initial value problems for a system of ordinary differential equation by means of characteristics[3]. A similar problem was solved by Sun[4] for the shock wave with small amplitudes. The general form of the solution can be expressed as

$$\begin{aligned} x - x_0 &= \left(p_x \frac{\partial g}{\partial p_x} + p_y \frac{\partial g}{\partial p_y}\right)^{-1} \frac{\partial g}{\partial p_x} t \\ y - y_0 &= \left(p_x \frac{\partial g}{\partial p_x} + p_y \frac{\partial g}{\partial p_y}\right)^{-1} \frac{\partial g}{\partial p_y} t \end{aligned} \tag{45}$$

where x_0 and y_0 indicate the initial position of the wave front at $t = 0$. The procedure for constructing the wave surface is quite straightforward. First, we determine the normal vector (n_x, n_y) for the initial wave front. Then, we substitute the values of n_x and n_y in the equation $|a_{ij}| = 0$ (or $|b_{ij}| = 0$) to obtain the corresponding normal velocity c_n for the wave front. From eqns (43) and (44) we obtain the values of p_x, p_y, $\partial g/\partial p_x$ and $\partial g/\partial p_y$ which are then used to determine the position of the wave front (x, y) at time t.

As an illustrative example, we consider the shock wave (or acceleration wave) with small amplitude propagating in a laminated plate under in-plane deformation. In this case, the transverse shear wave is uncoupled from the other modes (see eqn (24)), and we have

$$g(p_x, p_y) = \alpha_1 p_x^2 + \alpha_2 p_y^2 + 2\alpha_3 p_x p_y - P = 0 \tag{46}$$

where

$$\begin{aligned} \alpha_1 &= N_x + A_{55} \\ \alpha_2 &= N_y + A_{44} \\ \alpha_3 &= N_{xy} + A_{45} \end{aligned} \tag{47}$$

Substitution of eqn (46) in eqn (45) leads to

$$\begin{aligned} x - x_0 &= (\alpha_1 p_x + \alpha_3 p_y)t \\ x - y_0 &= (\alpha_3 p_x + \alpha_2 p_y)t \end{aligned} \tag{48}$$

If the wave front is generated from a point source at the origin, then $x_0 = y_0 = 0$. In this case, we can eliminate p_x and p_y from eqn (48) and (46) to obtain

$$x^2/\beta_1^2 + y^2/\beta_2^2 - 2\beta_3 xy = Pt^2 \tag{49}$$

where

$$\begin{aligned} \beta_1^2 &= \alpha_1 - \alpha_3^2/\alpha_2 \\ \beta_2^2 &= \alpha_2 - \alpha_3^2/\alpha_1 \\ \beta_3 &= \alpha_3/(\alpha_1\alpha_2 - \alpha_3^2) \end{aligned} \tag{50}$$

It is evident that eqn (49) represents an ellipse of which the major axis is inclined by an angle ϕ given by

$$\tan 2\phi = 2\alpha_3/(\alpha_1 - \alpha_2) \tag{51}$$

A photoelastic study of stress waves in laminated composites was carried out by Dally *et al.* [5]. It was found that the wave surface was in fact elliptical in shape. Some interesting discussions concerning wave surfaces were presented in [4].

If in eqn (49) we set $\alpha_3 = 0$, then the major axis of the ellipse coincides with the x-axis. Also eqn (49) reduces to

$$x^2/\alpha_1 + y^2/\alpha_2 = Pt^2 \tag{52}$$

It is obvious that if $\alpha_1 = \alpha_2$, which is identical to eqn (27), then the wave surface forms a circle. The reason is that under these conditions the wave front propagate with the same velocity in all directions.

From eqn (47), it is noted that α_1 and α_2 can approach zero if N_x and N_y are negative, or, in other words, compressive. Consider, for example, the case $N_x = N_0$ and $N_y = -N_0$. From eqn (52), it is seen that the ellipse of the wave front would have a long major axis and very short minor axis as the value of N_0 increases. That is, the compression in the y-direction reduces the wave propagation velocity in the respective direction. If $N_0 \geq A^*_{44}$, then the wave front can not propagate. This is also an indication that the under such compressive initial force, the plate may not be initially stable.

7. NUMERICAL RESULTS

A typical graphite-epoxy composite has the following engineering constants:

$$\begin{aligned} &E_L = 25 \times 10^6\,\text{psi},\ E_T = 1 \times 10^6\,\text{psi},\ G_{LT} = 0{\cdot}5 \times 10^6\,\text{psi} \\ &G_{TT} = 0{\cdot}2 \times 10^6\,\text{psi},\ \nu_{LT} = 0{\cdot}25,\ \nu_{TT} = 0{\cdot}25 \end{aligned} \tag{53}$$

where L and T are the directions parallel and normal to the fibers, respectively; ν_{LT} is the Poisson's ratio measuring the lateral strain under uniaxial normal stress parallel to the fibers, and ν_{TT} is the Poisson's ratio defined in the same manner. The corresponding reduced stiffnesses are given by

$$\{Q_{ij}\} = \begin{Bmatrix} 25{\cdot}062 & 0{\cdot}250 & 0 \\ 0{\cdot}250 & 1{\cdot}002 & 0 \\ 0 & 0 & 0{\cdot}5 \end{Bmatrix} \times 10^6\,\text{psi} \tag{54a}$$

$$Q_{44} = 0{\cdot}2 \times 10^6\,\text{psi},\quad Q_{55} = 0{\cdot}5 \times 10^6\,\text{psi},\quad Q_{45} = 0 \tag{54b}$$

The reduced stiffness coefficients for layers with fibers orienting in other directions can be obtained by the usual coordinate transformation law.

In Figs. 1 and 2, the velocities of shock waves of large amplitudes propagating in initially

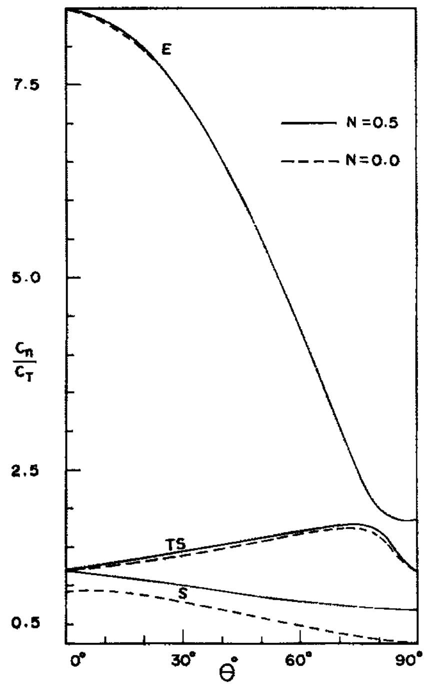

Fig. 1. Normal velocities vs θ for shock waves in a 0-laminate. $N = 2N_v/h(G_{TT} + G_{LT})$, $[\dot{w}]/c_T = 1$.

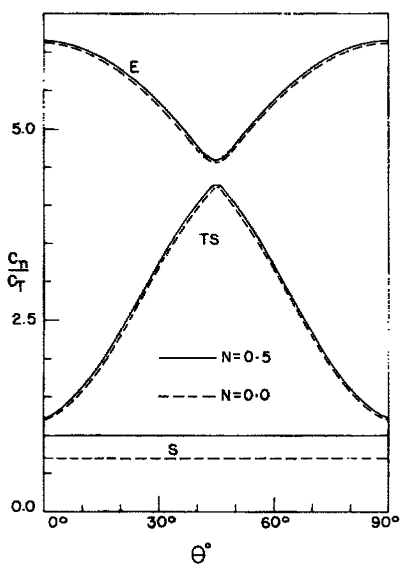

Fig. 2. Normal velocities vs θ for shock waves in a 0-90-90-0-laminate, $[\dot{w}]/c_T = 1$.

stressed laminates are plotted versus the direction of propagation θ. The plates considered are cross-ply laminates symmetrically stacked in order to reduce the bending and extension coupling. It should be noted that there are two factors which can affect the shock velocity, namely the amplitude of the shock, $[\dot{w}]$, and the initial in-plane force, $(N_v)_a$. It is clearly shown by the figures that both the amplitude and the initial force have considerable effect on the transverse shear shock wave. It is interesting to note that the effects of the amplitude and the initial force are opposite, i.e., a larger amplitude reduces the transverse shear shock velocity while a larger initial tensile force increases it.

The dependence of the shock velocity on the initial stress is shown in Fig. 3 for $[\dot{w}]/c_T = 1$ and $\theta = 30°$ for 0 and 0-90-90-0 laminates. Again the extension and twisting shear modes appear to be little affected by the presence of the initial stress, while there is substantial stiffening effect on the

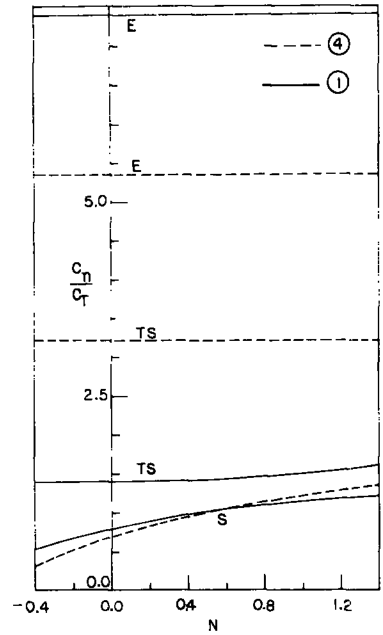

Fig. 3. Effect of the initial force N_ν on the normal velocities for 0-90-90-0- and 0-laminates at $\theta = 30°$ and $[\dot{w}]/c_T = 1. N = 2N_\nu/h(G_{TT} + G_{LT})$.

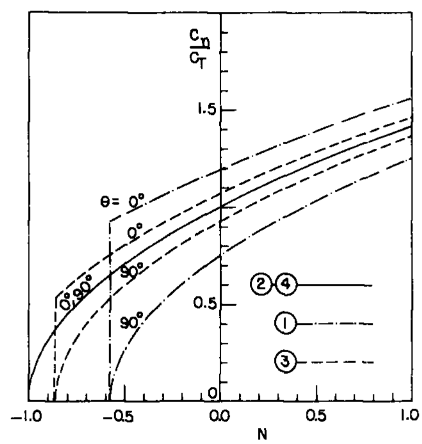

Fig. 4. Effect of N_ν on the shock velocity of the transverse shear mode for 0-90-90-0, 0-90-0, 0-90, and 0-laminates with $[\dot{w}] \approx 0$.

transverse shear mode. In Fig. 4, the influence of the in-plane force on the velocity of the transverse shear shock is presented for various laminates at different directions. It should be observed that the shock wave ceases to propagate at some critical compressive forces. The critical value is direction dependent. If the wave surface forms an enclosed contour, then the smallest value of N_ν in all directions should be considered as the maximum compressive force the plate can sustain.

In Figs. 5–10, the numerical results corresponding to the initial deformation given by eqn (30) as discussed in Section 5 are presented. In addition to the transverse shear mode, the other modes are affected by the initial deflection appreciably. For the symmetric laminates, however, the bending and twisting moments modes, which are not coupled with the other modes, agree with the linear solutions (without initial deformations). It is found that the influence of the initial deflection in fact is greater on the extension and twisting shear modes than on the transverse shear mode. This fact is revealed in Figs. 9 and 10.

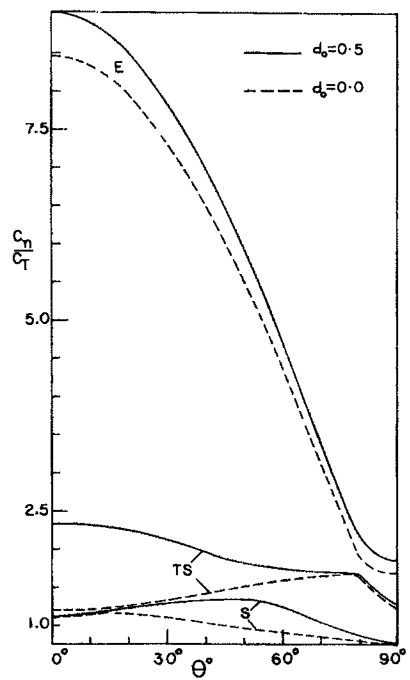

Fig. 5. Normal velocities of the wave fronts vs θ for an initially deflected 0-laminate with $\theta_0 = 45°$ and $[\dot{w}] \approx 0$.

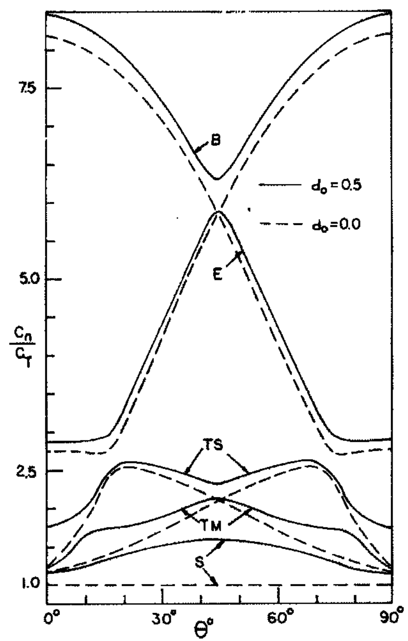

Fig. 6. Normal velocities of the wave fronts vs θ for an initially deflected 0-90-laminate with $\theta_0 = 45°$ and $[\dot{w}] \approx 0$.

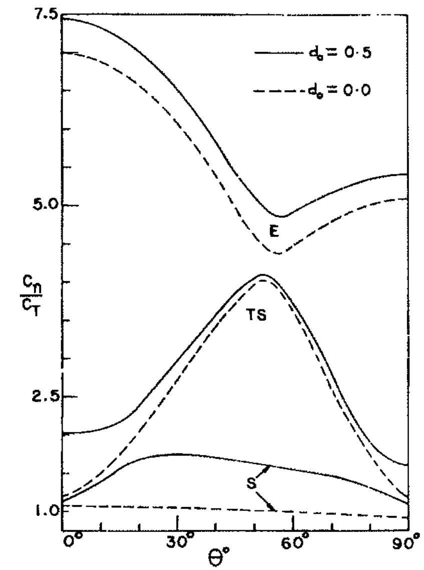

Fig. 7. Normal velocities of the wave fronts vs θ for an initially deflected 0-90-0-laminate with $\theta_0 = 45°$ and $[\dot{w}] \approx 0$.

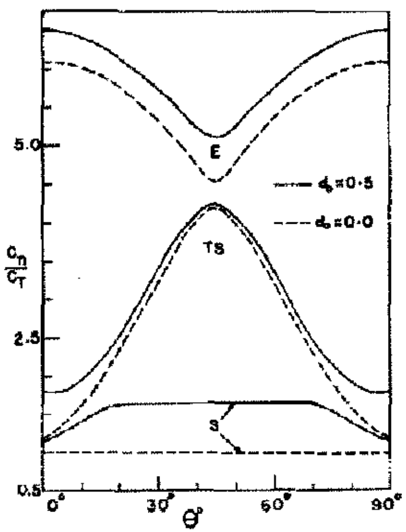

Fig. 8. Normal velocities of the wave fronts vs θ for an initially deflected 0-90-90-0-laminate with $\theta_0 = 45°$ and $[\dot{w}] = 0$.

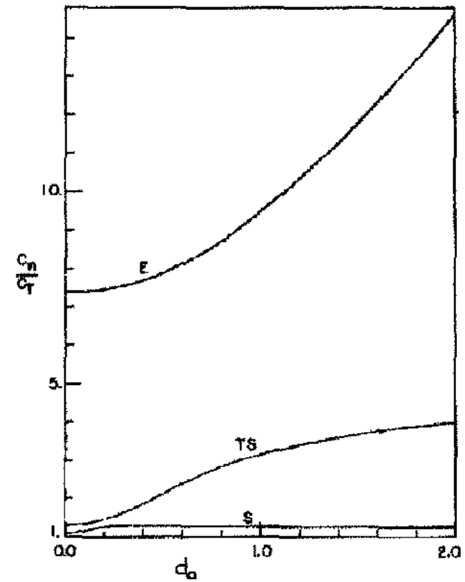

Fig. 9. Effect of the initial deflection on the velocities of the wave fronts for a 0-laminate with $\theta_0 = 45°$ and $\theta = 30°$.

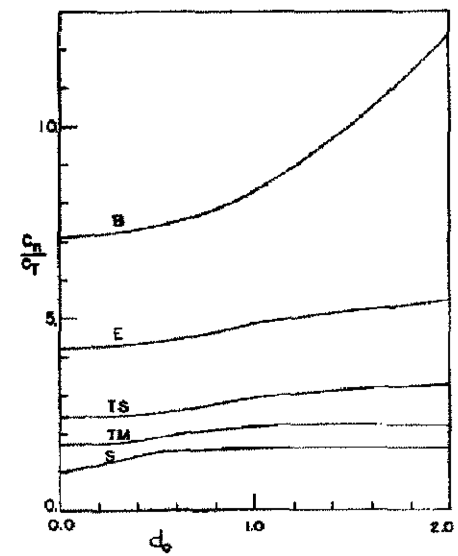

Fig. 10. Effect of the initial deflection on the velocities of the wave fronts for a 0-90-laminate with $\theta_0 = 45°$ and $\theta = 30°$.

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